Lesson 9: Product Rule

PRODUCT RULE: $\boxed{\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)}$

Warmup (with Product Rule):

$$y = x^2(x+1)$$

$$y' = \underset{\frac{d}{dx}[x^2]}{\underset{\uparrow}{2x}}(x+1) + x^2\underset{\frac{d}{dx}[x+1]}{\underset{\uparrow}{(1)}} = 2x^2 + 2x + x^2 = 3x^2 + 2x$$

(It's not always faster to use product rule.)

Ex 1 (a) $y = 3e^x \sin x$ $\quad \frac{d}{dx}[3e^x] = 3e^x \quad \frac{d}{dx}[\sin x] = \cos x$

$$y' = 3e^x \sin x + 3e^x \cos x = 3e^x(\sin x + \cos x)$$

(b) $y = \sin^2 x = \sin x \cdot \sin x$ $\quad \frac{d}{dx}[\sin x] = \cos x \quad \frac{d}{dx}[\sin x] = \cos x$

$$y' = \cos x \sin x + \sin x \cos x$$

$$= 2\sin x \cos x$$

(c) $y = -x(\cos x + e^x)$ $\quad \frac{d}{dx}[-x] = -1 \quad \frac{d}{dx}[\cos x + e^x] = -\sin x + e^x$

$$y' = -1(\cos x + e^x) + (-x)(-\sin x + e^x)$$

Ex 2 Find $y'(8)$ if $y = \frac{\sin x}{\sqrt[3]{x}} = (\sin x)(x^{-1/3})$

$\frac{d}{dx}[\sin x] = \cos x \quad \frac{d}{dx}[x^{-1/3}] = -\frac{1}{3}x^{-4/3}$

$$y' = \cos x(x^{-1/3}) + \sin x\left(-\frac{1}{3}x^{-4/3}\right)$$

$$y'(8) = \cos 8\,(8^{-1/3}) + \sin 8\left(-\frac{1}{3}(8)^{-4/3}\right)$$

$$= \frac{1}{2}\cos 8 + \sin 8\left(-\frac{1}{3}\right)\left(\frac{1}{16}\right)$$

$$= \boxed{\frac{1}{2}\cos 8 - \frac{1}{48}\sin 8}$$